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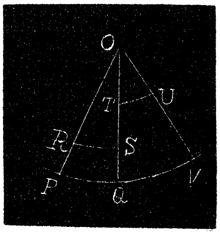
39. Proposed by B. F. FINKEL, A. M., Professor of Mathematics and Physics in Drury College, Springfield, Missouri.

A person whose height is a and weight W stands in a swing whose length is l. Supposing the initial inclination of the swing to the vertical is α and that the person always crouches when in the highest position and stands up when in the lowest, his center of gravity moving through a distance b measured from lower part of swing upward, find how much the arc is increased after n complete vibrations.

I. Solution by O. W. ANTHONY, M. Sc., Professor of Mathematics, Columbian University, Washington, D. C.

Let RS be the path of the center of gravity from extreme position to vertical, TU the path from vertical to other extreme position.

$OP=l$, $RP=k$, (say), $TS=b$; $\angle ROS=a$, $\angle TOU=\alpha_1$, etc.



The energy acquired by swing in passing from R to S is $(l-k)(1-\cos\alpha)W$.

When it has passed to V the energy is $(l-k-b)(1-\cos\alpha_1)W$.

By conservation of energy

$$(l-k-b)(1-\cos\alpha_1)W=(l-k)(1-\cos\alpha)W.$$

Whence $1-\cos\alpha_1=\dfrac{l-k}{l-k-b}(1-\cos\alpha)$.

In passing back to original position

$$1-\cos\alpha_2=\frac{l-k}{l-k-b}(1-\cos\alpha_1)=\left(\frac{l-k}{l-k-b}\right)^2(1-\cos\alpha).$$

For two complete vibrations $1-\cos\alpha_4=\left(\dfrac{l-k}{l-k-b}\right)^4(1-\cos\alpha)$.

In like manner for n complete vibrations

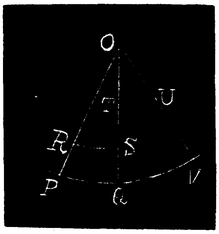
$$1-\cos\alpha_{2n}=\left(\frac{l-k}{l-k-b}\right)^{2n}(1-\cos\alpha) \text{ or } \sin(\tfrac{1}{2}\alpha_{2n})=\left(\frac{l-k}{l-k-b}\right)^{n}\sin(\tfrac{1}{2}\alpha),$$

which enables us to compute the increase in amplitude.

II. Solution by the PROPOSER.

Let O be the point of suspension of the swing, S the position of the center of gravity of the man when crouching, and T its position when the man is standing, and Q the lower end of the swing.

Let $OQ=l$, $SQ=k$, the distance from lower end of swing to the center of gravity of the man when he is crouching, $ST=b$, $\angle QOP=\alpha$, and $\angle QOV=\theta_0$.



Then the velocity, v, of the man at the point $Q,=\sqrt{2gl(l-k)(1-\cos\alpha)}$, *Bowser's Analytical Mechanics*, page 350, Article 192. Hence, his energy due to his weight $=\dfrac{W}{2g}v^2=W(l-k)(1-\cos\alpha)$. This energy will carry him to V and equals $W(l-k-b)(1-\cos\theta_0)$, since the man rises at the point Q. Since he crouches at the point V, his energy at the point Q on his return is $W(l-k)(1-\cos\theta_0)$. This energy will carry him to a point to the left of S, and the energy expended will be $W(l-k-b)(1-\cos\theta_1)$, where θ_1 is the angle between the vertical and the swing.

According to the principle of the conservation of energy, we have,

$$W(l-k)(1-\cos\alpha)=W(l-k-b)(1-\cos\theta_1)\ldots\ldots\ldots\ldots\ldots(1),$$

$$W(l-k)(1-\cos\theta_0)=W(l-k-b)(1-\cos\theta_2)\ldots\ldots\ldots\ldots\ldots(2),$$

$$W(l--k)(1-\cos\theta_1)=W(l-k-b)(1-\cos\theta_3)\ldots\ldots\ldots\ldots\ldots(3),$$

$$\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot$$

$$W(l-k)(1-\cos\theta_{2n-1})=W(l-k-b)(1-\cos\theta_{2n-1})\ldots\ldots(2n).$$

Multiplying these equations together member for member, and solving for $1-\cos\theta_{2n}$, we have,

$$1-\cos\theta_{2n}=\left(\frac{l-k}{l-k-b}\right)^{2n}(1-\cos\alpha),\text{ or }\sin^2(\tfrac{1}{2}\theta_{2n})=\left(\frac{l-k}{l-k-b}\right)^{2n}(\sin^2\tfrac{1}{2}\alpha).$$

Whence, $\sin\frac{1}{2}\theta_{2n}=\left(\dfrac{l-k}{l-k-b}\right)^{n}\sin\frac{1}{2}\alpha$, or $\theta_{2n}=2\sin^{-1}\left[\left(\dfrac{l-k}{l-k-b}\right)\sin\frac{1}{2}\alpha\right]$.

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DIOPHANTINE ANALYSIS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

42. Proposed by E. B. ESCOTT, Chicago, Illinois.

Required all the parallelograms whose sides a, b and diagonals c, d, are rational.